Collective dynamics in nonidentical oscillators with indirect coupling

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The study of coupled oscillators is one of the most fundamental problems in nonlinear dynamics with applications in a variety of physical and biological fields. Many synchronization phenomena, such as full, cluster, phase and lag synchronization, etc. and some coupling configurations, such as star, lattice, fully connected and nearest neighbor coupling etc. for identical coupled oscillators have been extensively studied. In practice, it is inevitable that the oscillators are not strictly identical, therefore, considerable attention has been devoted to the problem of the persistence of synchronization in networks of identical oscillators when some mismatch terms are introduced. Mismatching effects on synchronization of coupled Rossler oscillators, skew tent maps and Lorenz systems were investigated.

However, most of the previous analysis on synchronization phenomena and coupling configurations was limited to direct coupling, i.e. any two coupled oscillators can interact directly. While, indirect interaction is crucial for coordinated behaviors in multicellular systems, such asintracellular signal processing, pattern formation and various social behaviors in bacteria . It is accomplished through a diffusive and mixing process with a common outside environment. Collective dynamics of interacting multicellular systems has been confirmed by some fundamental experiments. Moreover, several theoretical models based on indirect coupling, i.e. coupled repressilators, genetic relaxation oscillators and cell-ensemble model for glycolytic oscillations have been successfully established to examine the collective dynamics and its basic mechanism. The indirect coupling process uses transmembrane diffusion of signal molecules or metabolites through the extracellular medium, by which multicellular system can regulate gene expression collectively and therefore control behaviors on a community-wide scale.

Although collective dynamics in a population of oscillators with indirect coupling has been confirmed by experiments and theoretical models, the existence and stability of such synchronization phenomena are not predicted and studied theoretically. Moreover, individual cells exhibit cell-cell variation in their period lengths and small diversity is always in reality. Accordingly, it seems natural to consider the collective dynamics in a population of nonidentical and noisy oscillators with indirect coupling. The approach we present here examines synchronization in a population of nonidentical oscillators interacting by exchange of signal molecules through the extracellular medium, which is more biologically plausible. A rigorous criteria is presented via the Lyapunov function method, under which the global stability of the synchronous motion can be guaranteed and the synchronization error, owing to the mismatch terms of the nonidentical oscillators, can be estimated.

[1]: Ruiqi Wang and Luonan Chen, "Synchronizing genetic oscillators by signaling molecules" Journal of biological rhythms, Vol. 20, No. 3, 257-269, 2005.
[2] Luonan Chen, Ruiqi Wang, Tianshou Zhou, KazuyukiAihara,"Noise-induced cooperative behavior in a multicell system", Bioinformatics, Vol. 21, 2722–2729, 2005.